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THE GEOMETRY OF QUADRATIC DIFFERENTIAL SYSTEMS WITH A WEAK FOCUS OF SECOND ORDER

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Planar quadratic differential systems occur in many areas of applied mathematics. Although more than one thousand papers were written on these systems, a complete understanding of this class is still missing. Classical problems, and in particular, Hilbert's 16th problem [Hilbert, 1900, 1902], are still open for this class. In this article we make an interdisciplinary global study of the subclass $\overline{QW2}$ which is the closure within real quadratic differential systems, of the family QW2 of all such systems which have a weak focus of second order. This class $\overline{QW2}$ also includes the family of all quadratic differential systems possessing a weak focus of third order and topological equivalents of all quadratic systems with a center.

The bifurcation diagram for this class, done in the adequate parameter space which is the three-dimensional real projective space, is quite rich in its complexity and yields 373 subsets with 126 phase portraits for $\overline{QW2}$, 95 for QW2, 20 having limit cycles but only three with the maximum number of limit cycles (two) within this class. The phase portraits are always represented in the Poincaré disc. The bifurcation set is formed by an algebraic set of bifurcations of singularities, finite or infinite and by a set of points which we suspect to be analytic corresponding to global separatrices which have connections.

Algebraic invariants were needed to construct the algebraic part of the bifurcation set, symbolic computations to deal with some quite complex invariants and numerical calculations to determine the position of the analytic bifurcation set of connections.

The global geometry of this class $\overline{QW2}$ reveals interesting bifurcations phenomena; for example, all phase portraits with limit cycles in this class can be produced by perturbations of symmetric (reversible) quadratic systems with a center. Many other nonlinear phenomena displayed here form material for further studies.

Keywords: Quadratic differential systems; weak focus; limit cycle; phase portraits; bifurcation diagram.