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Geometric configurations of singularities for quadratic differential systems with total finite multiplicity lower than 2

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Abstract. In [3] we classified globally the configurations of singularities at infinity of quadratic differential systems, with respect to the *geometric equivalence relation*. The global classification of configurations of finite singularities was done in [2] modulo the coarser *topological equivalence relation* for which no distinctions are made between a focus and a node and neither are they made between a strong and a weak focus or between foci of different orders. These distinctions are however important in the production of limit cycles close to the foci in perturbations of the systems. The notion of *geometric equivalence relation* of configurations of singularities allows us to incorporates all these important purely algebraic features. This equivalence relation is also finer than the *qualitative equivalence relation* introduced in [20]. In this article we initiate the joint classification of configurations of singularities, finite and infinite, using the finer *geometric equivalence relation*, for the subclass of quadratic differential systems possessing finite singularities of total multiplicity $m_f \leq 1$. We obtain 84 *geometrically distinct* configurations of singularities for this family. We also give here the global bifurcation diagram, with respect to the *geometric equivalence relation*, of configurations of singularities, both finite and infinite, for this class of systems. This bifurcation set is algebraic. The bifurcation diagram is done in the 12-dimensional space of parameters and it is expressed in terms of polynomial invariants. The results can therefore be applied for any family of quadratic systems, given in any normal form. Determining the configurations of singularities for any family of quadratic systems, becomes thus a simple task using computer algebra calculations.

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1 Introduction and statement of main results

We consider here differential systems of the form

$$\frac{dx}{dt} = p(x, y), \qquad \frac{dy}{dt} = q(x, y), \tag{1}$$

where $p, q \in \mathbb{R}[x, y]$, i.e. p, q are polynomials in x, y over $\mathbb{R}$. We call *degree* of a system (1) the integer $m = \max(\deg p, \deg q)$. In particular we call *quadratic* a differential system (1) with $m = 2$. We denote here by **QS** the whole class of real quadratic differential systems.