

# Geometric configurations of singularities for quadratic differential systems with three distinct real simple finite singularities

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*To Professor Yvonne Choquet-Bruhat*

**Abstract.** In this work we classify, with respect to the *geometric equivalence relation*, the global configurations of singularities, finite and infinite, of quadratic differential systems possessing exactly three distinct finite simple singularities. This relation is finer than the *topological equivalence relation* which does not distinguish between a focus and a node or between a strong and a weak focus or between foci (or saddles) of different orders. Such distinctions are, however, important in the production of limit cycles close to the foci (or loops) in perturbations of the systems. The notion of *geometric equivalence relation* of configurations of singularities allows us to incorporate all these important geometric features which can be expressed in purely algebraic terms. The *geometric classification* of all configurations of singularities, finite and infinite, of quadratic systems was initiated in a work published in 2013 when the classification was done for systems with total multiplicity  $m_f$  of finite singularities less than or equal to one. That work was continued in an article which is due to appear in 2014 where the geometric classification of configurations of singularities was done for the case  $m_f = 2$ . In this article we go one step further and obtain the *geometric classification* of singularities, finite and infinite, for the subclass mentioned above. We obtain 147 *geometrically distinct* configurations of singularities for this family. We give here the global bifurcation diagram of configurations of singularities, both finite and infinite, with respect to the *geometric equivalence relation*, for this class of systems. The bifurcation set of this diagram is algebraic. The bifurcation diagram is done in the 12-dimensional space of parameters and it is expressed in terms of polynomial invariants, a fact which gives us an algorithm for determining the geometric configuration of singularities for any quadratic system in this particular class.

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