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FROM TOPOLOGICAL TO GEOMETRIC EQUIVALENCE IN THE CLASSIFICATION OF SINGULARITIES AT INFINITY FOR QUADRATIC VECTOR FIELDS

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ABSTRACT. In the topological classification of phase portraits no distinctions are made between a focus and a node and neither are they made between a strong and a weak focus or between foci of different orders. These distinctions are however important in the production of limit cycles close to the foci in perturbations of the systems. The distinction between the one direction node and the two directions node, which plays a role in understanding the behavior of solution curves around the singularities at infinity, is also missing in the topological classification.

In this work we introduce the notion of *geometric equivalence relation of singularities* which incorporates these important purely algebraic features. The *geometric* equivalence relation is finer than the *topological* one and also finer than the *qualitative equivalence relation* introduced in [**19**]. We also list all possibilities we have for finite and infinite singularities, taking into consideration these finer distinctions, and introduce notation for each one of them.

In this work we give the classification theorem and bifurcation diagram in the 12-dimensional parameter space, using the *geometric equivalence relation*, of the class of quadratic systems according to the configuration of singularities at infinity of the systems. Our classification theorem, stated in terms of invariant polynomials, is an algorithm for computing the *geometric configurations* of infinite singularities for any family of quadratic systems, in any normal form.

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