

## A universal constant for semistable limit cycles

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**Abstract.** We consider one-parameter families of 2-dimensional vector fields  $X_{\mu}$  having in a convenient region *R* a semistable limit cycle of multiplicity 2m when  $\mu = 0$ , no limit cycles if  $\mu \leq 0$ , and two limit cycles one stable and the other unstable if  $\mu \gtrsim 0$ .

We show, analytically for some particular families and numerically for others, that associated to the semistable limit cycle and for positive integers *n* sufficiently large there is a power law in the parameter  $\mu$  of the form  $\mu_n \approx Cn^{\alpha} < 0$  with  $C, \alpha \in \mathbb{R}$ , such that the orbit of  $X_{\mu_n}$  through a point of  $p \in R$  reaches the position of the semistable limit cycle of  $X_0$  after given *n* turns.

The exponent  $\alpha$  of this power law depends only on the multiplicity of the semistable limit cycle, and is independent of the initial point  $p \in R$  and of the family  $X_{\mu}$ . In fact  $\alpha = -2m/(2m-1)$ . Moreover the constant *C* is independent of the initial point  $p \in R$ , but it depends on the family  $X_{\mu}$  and on the multiplicity 2m of the limit cycle  $\Gamma$ .

## Mathematical subject classification: 58F14, 58F21, 58F30.

Key words: semistable limit cycle, semistable fixed point, universal constant, power law.

<sup>#</sup>CAM-233/10. Received: 06/VII/10. Accepted: 18/XI/10.

<sup>\*</sup>The first two authors are partially supported by a MEC/FEDER grant BFM2008-03437, and a CIRIT grant number 2009SGR 410. The third author is partially supported by a grant FAPESP-2007/06896-5. All authors are also supported by the joint project CAPES-MECD grant HBP-2009-0025-PC.