DYNAMICS OF THE HIGGINS–SELKOV AND SELKOV SYSTEMS

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Abstract. We describe the global dynamics in the Poincaré disc of the Higgins–Selkov model

$$x' = k_0 - k_1 xy^2, \quad y' = -k_2 y + k_1 xy^2,$$

where k_0, k_1, k_2 are positive parameters, and of the Selkov model

$$x' = -x + ay + x^2 y, \quad y' = b - ay - x^2 y,$$

where a, b are positive parameters.

1. Introduction and statement of the results

The Higgins–Selkov model of glycolysis is

$$\dot{x} = k_0 - k_1 xy^2, \quad \dot{y} = k_1 xy^2 - k_2 y, \tag{1}$$

where the unknowns x and y are concentrations which are non-negative and k_i for $i = 0, 1, 2$ are the reaction positive constants, see [8] for the biological details of this model.

We will describe the global dynamics of the differential system (1) in the Poincaré disc for all positive values of k_0, k_1 and k_2. For a definition of the Poincaré disc, and of its separatrices and canonical regions see subsection 3.6 of the appendix. We denote by S (respectively R) the number of separatrices (respectively canonical regions) of a phase portrait in the Poincaré disc. Thus our first main result is:

Theorem 1. *The Higgins–Selkov system* (1)*, after a rescaling of its variables, can be written as*

$$x' = 1 - xy^2, \quad y' = ay(xy - 1), \tag{2}$$

with $a > 0$. The global phase portraits of this system for $a \in \mathbb{R}$ is topologically equivalent to the one of

- *Figure 1(A) for $a < 0$, with $S = 19$, $R = 6$;*
- *Figure 1(B) for $a = 0$, with $S = \infty$;*
- *Figure 1(C) for $a \in (0, 1]$, with $S = 17$, $R = 4$;*

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