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# QUADRATIC SYSTEMS WITH A RATIONAL FIRST INTEGRAL OF DEGREE 2: A COMPLETE CLASSIFICATION IN THE COEFFICIENT SPACE $\mathbb{R}^{12}$

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A quadratic polynomial differential system can be identified with a single point of $\mathbb{R}^{12}$ through the coefficients. Using the algebraic invariant theory we classify all the quadratic polynomial differential systems of $\mathbb{R}^{12}$ having a rational first integral of degree 2. We show that there are only 24 topologically different phase portraits in the Poincaré disc associated to this family of quadratic systems up to a reversal of the sense of their orbits, and we provide a unique representative of every class modulo an affine change of variables and a rescalling of the time variable. Moreover, each one of these 24 representatives is determined by a set of invariant conditions and each respective first integral is given in invariant form directly in $\mathbb{R}^{12}$.

## 1. Introduction.

Let $P$ and $Q$ be two real polynomials in the variables $x$ and $y$, then we say that

$$\dot{x} = \frac{dx}{dt} = P(x, y), \qquad \dot{y} = \frac{dy}{dt} = Q(x, y),$$

is a *quadratic polynomial differential system* or simply a *quadratic system* if the maximum of the degrees of the polynomials $P$ and $Q$ is two. Associated to this quadratic system we have its corresponding *quadratic polynomial vector field* $X = (P, Q) : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, or simply a *quadratic vector field.*

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