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Quadratic systems with a polynomial first integral: A complete classification in the coefficient space $\mathbb{R}^{12}$

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ABSTRACT

In this paper we are going to apply the invariant theory to give invariant conditions on the coefficients of any non-degenerate quadratic system in order to determine if it has or not a polynomial first integral without using any normal form. We obtain that the existence of polynomial first integral is directly related with the fact that all the roots of a convenient cubic polynomial are rational and negative. The coefficients of this cubic polynomial are invariants related with some geometric properties of the system.

1. Introduction and the statement of the main result

Let $\mathbb{R}[x, y]$ be the ring of all polynomials in the variables x and y with coefficients in $\mathbb{R}$. In this paper we deal with *quadratic polynomial differential systems* in $\mathbb{R}^2$ of the form

$$\frac{dx}{dt} = x' = P(x, y), \qquad \frac{dy}{dt} = y' = Q(x, y), \tag{1}$$

where $P, Q \in \mathbb{R}[x, y]$ and $\max\{\deg P, \deg Q\} = 2$. In what follows such differential systems will be called simply *quadratic systems*.

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