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Quadratic systems with a rational first integral of degree three: a complete classification in the coefficient space \mathbb{R}^{12}

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Abstract. A quadratic polynomial differential system can be identified with a single point of \mathbb{R}^{12} through its coefficients. The phase portrait of the quadratic systems having a rational first integral of degree 3 have been studied using normal forms. Here using the algebraic invariant theory, we characterize all the non-degenerate quadratic polynomial differential systems in \mathbb{R}^{12} having a rational first integral of degree 3. We show that there are only 31 different topological phase portraits in the Poincaré disc associated to this family of quadratic systems up to a reversal of the sense of their orbits, and we provide representatives of every class modulo an affine change of variables and a rescaling of the time variable. Moreover, each one of these 31 representatives is determined by a set of algebraic invariant conditions and we provide for it a first integral.

Keywords Quadratic vector fields · Integrability · Rational first integral · Phase portraits

Mathematics Subject Classification (2000) 34C05 · 34A34

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