# Joan C. Artés • Jaume Llibre • <br> Nicolae Vulpe <br> Quadratic systems with a rational first integral of degree three: a complete classification in the coefficient space $\mathbb{R}^{12}$ 


#### Abstract

A quadratic polynomial differential system can be identified with a single point of $\mathbb{R}^{12}$ through its coefficients. The phase portrait of the quadratic systems having a rational first integral of degree 3 have been studied using normal forms. Here using the algebraic invariant theory, we characterize all the non-degenerate quadratic polynomial differential systems in $\mathbb{R}^{12}$ having a rational first integral of degree 3 . We show that there are only 31 different topological phase portraits in the Poincaré disc associated to this family of quadratic systems up to a reversal of the sense of their orbits, and we provide representatives of every class modulo an affine change of variables and a rescaling of the time variable. Moreover, each one of these 31 representatives is determined by a set of algebraic invariant conditions and we provide for it a first integral.


Keywords Quadratic vector fields • Integrability • Rational first integral • Phase portraits

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