Electronic Journal of Differential Equations, Vol. 2012 (2012), No. 09, pp. 1–35.
ISSN: 1072-6691. URL: http://ejde.math.txstate.edu or http://ejde.math.unt.edu
ftp ejde.math.txstate.edu

COMPLETE GEOMETRIC INVARIANT STUDY OF TWO CLASSES OF QUADRATIC SYSTEMS

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Abstract. In this article, using affine invariant conditions, we give a complete study for quadratic systems with center and for quadratic Hamiltonian systems. There are two improvements over the results in [30] that studied centers up to GL-invariant, and over the results in [1] that classified Hamiltonian quadratic systems without invariants. The geometrical affine invariant study presented here is a crucial step toward the goal of the invariant classification of all quadratic systems according to their singularities, finite and infinite.

1. Introduction and statement of results

Let $\mathbb{R}[x, y]$ be the ring of the polynomials in the variables x and y with coefficients in $\mathbb{R}$. We consider a system of polynomial differential equations, or simply a polynomial differential system, in $\mathbb{R}^2$ defined by

$$\begin{aligned} \dot{x} &= P(x, y), \\ \dot{y} &= Q(x, y), \end{aligned} \tag{1.1}$$

where $P, Q \in \mathbb{R}[x, y]$. We say that the maximum of the degrees of the polynomials P and Q is the degree of system (1.1). A *quadratic polynomial differential system* or simply a *quadratic system* (QS) is a polynomial differential system of degree 2. We say that the quadratic system (1.1) is *non-degenerate* if the polynomials P and Q are relatively prime or coprime; i.e., g.c.d. $(P, Q) = 1$.

During the previous one-hundred years quadratic vector fields have been investigated intensively as one of the easiest but far from trivial families of nonlinear differential systems, and more than one thousand papers have been published about these vectors fields (see for instance [24, 33, 32]). However, the problem of classifying all the quadratic vector fields (even integrable ones) remains open. For more information on the integrable differential vector fields in dimension 2, see for instance [9, 18].

Poincaré [23] defined the notion of a *center* for a real polynomial differential system in the plane (i.e. an isolated singularity surrounded by periodic orbits). The analysis of the limit cycles which bifurcate from a focus or a center of a quadratic system was made by Bautin [7], by providing the structure of the power series development of the displacement function defined near a focus or a center

2000 *Mathematics Subject Classification.* 34C05, 34A34.
Key words and phrases. Quadratic vector fields; weak singularities; type of singularity.

Submitted May 16, 2011. Published January 13, 2012.