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Quadratic systems with an integrable saddle: A complete classification in the coefficient space $\mathbb{R}^{12}$

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ABSTRACT

A quadratic polynomial differential system can be identified with a single point of $\mathbb{R}^{12}$ through the coefficients. Using the algebraic invariant theory we classify all the quadratic polynomial differential systems of $\mathbb{R}^{12}$ having an integrable saddle. We show that there are only 47 topologically different phase portraits in the Poincaré disk associated to this family of quadratic systems up to a reversal of the sense of their orbits. Moreover each one of these 47 representatives is determined by a set of affine invariant conditions.

1. Introduction and statement of main results

Let $\mathbb{R}[x, y]$ be the ring of the polynomials in the variables x and y with coefficients in $\mathbb{R}$. We consider a system of polynomial differential equations or simply a *polynomial differential system* in $\mathbb{R}^2$ defined by

$$\begin{aligned} \dot{x} &= P(x, y), \\ \dot{y} &= Q(x, y), \end{aligned} \tag{1}$$

where $P, Q \in \mathbb{R}[x, y]$. We say that the maximum of the degrees of the polynomials P and Q is the *degree* of system (1). A *quadratic polynomial differential system* or simply a *quadratic system* (QS) is a polynomial differential system of degree 2. We say that a quadratic system (1) is *non-degenerate* if the polynomials P and Q are relatively prime.

In [1] Poincaré defined the notion of a *center* for a real polynomial differential system in the plane (i.e. an isolated singularity surrounded by a continuum of periodic orbits). Now, using a linear change of coordinates and a rescaling of the independent variable, we transform any polynomial differential system having a focus at the origin with purely imaginary eigenvalues (i.e. having a *weak focus*) or a center into the form

$$\begin{aligned} \dot{x} &= p(x, y) = y + p_2(x, y) + \cdots + p_m(x, y), \\ \dot{y} &= q(x, y) = -x + q_2(x, y) + \cdots + q_m(x, y), \end{aligned} \tag{2}$$

where

$$p_i(x, y) = \sum_{j=0}^{i} a_{ij} x^{i-j} y^j, \qquad q_i(x, y) = \sum_{j=0}^{i} b_{ij} x^{i-j} y^j.$$

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