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Topological Classification of Quadratic Polynomial Differential Systems with a Finite Semi-Elemental Triple Saddle

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The study of planar quadratic differential systems is very important not only because they appear in many areas of applied mathematics but due to their richness in structure, stability and questions concerning limit cycles, for example. Even though many papers have been written on this class of systems, a complete understanding of this family is still missing. Classical problems, and in particular Hilbert's 16th problem [Hilbert, 1900, 1902], are still open for this family. In this article, we make a global study of the family $\mathbf{QT\overline{S}}$ of all real quadratic polynomial differential systems which have a finite semi-elemental triple saddle (triple saddle with exactly one zero eigenvalue). This family modulo the action of the affine group and time homotheties is three-dimensional and we give its bifurcation diagram with respect to a normal form, in the three-dimensional real space of the parameters of this normal form. This bifurcation diagram yields 27 phase portraits for systems in $\mathbf{QT\overline{S}}$ counting phase portraits with and without limit cycles. Algebraic invariants are used to construct the bifurcation set and we present the phase portraits on the Poincaré disk. The bifurcation set is not just algebraic due to the presence of a surface found numerically, whose points correspond to connections of separatrices.

Keywords: Quadratic differential systems; semi-elemental triple saddle; phase portraits; bifurcation diagram; algebraic invariants.