Journal of Dynamics and Differential Equations
https://doi.org/10.1007/s10884-020-09871-2

Structurally Unstable Quadratic Vector Fields of Codimension Two: Families Possessing Either a Cusp Point or Two Finite Saddle-Nodes

Joan C. Artés[1] · Regilene D. S. Oliveira[2] · Alex C. Rezende[3]

Received: 23 January 2019 / Revised: 23 January 2019

Abstract
The goal of this paper is to contribute to the classification of the phase portraits of planar quadratic differential systems according to their structural stability. Artés et al. (Mem Am Math Soc 134:639, 1998) proved that there exist 44 structurally stable topologically distinct phase portraits in the Poincaré disc modulo limit cycles in this family, and Artés et al. (Structurally unstable quadratic vector fields of codimension one, Springer, Berlin, 2018) showed the existence of at least 204 (at most 211) structurally unstable topologically distinct phase portraits of codimension-one quadratic systems, modulo limit cycles. In this work we begin the classification of planar quadratic systems of codimension two in the structural stability. Combining the sets of codimension-one quadratic vector fields one to each other, we obtain ten new sets. Here we consider set AA obtained by the coalescence of two finite singular points, yielding either a triple saddle, or a triple node, or a cusp point, or two saddle-nodes. We obtain all the possible topological phase portraits of set AA and prove their realization. We got 34 new topologically distinct phase portraits in the Poincaré disc modulo limit cycles. Moreover, in this paper we correct a mistake made by the authors in the book of Artés et al. (Structurally unstable quadratic vector fields of codimension one, Springer, Berlin, 2018) and we reduce to 203 the number of topologically distinct phase portrait of codimension one modulo limit cycles.

Keywords Quadratic differential systems · Structural stability · Codimension two · Phase portrait · Saddle-node

✉ Alex C. Rezende
alexcr@ufscar.br

Joan C. Artés
artes@mat.uab.cat

Regilene D. S. Oliveira
regilene@icmc.usp.br

[1] Departament de Matemàtiques, Universitat Autònoma de Barcelona, Bellaterra, Spain

[2] Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, São Carlos, Brazil

[3] Departamento de Matemática, Universidade Federal de São Carlos, São Carlos, Brazil

Published online: 23 July 2020