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The Geometry of Quadratic Polynomial Differential Systems with a Finite and an Infinite Saddle-Node (A, B)

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Planar quadratic differential systems occur in many areas of applied mathematics. Although more than one thousand papers have been written on these systems, a complete understanding of this family is still missing. Classical problems, and in particular, Hilbert's 16th problem [Hilbert, 1900, 1902], are still open for this family. In this paper, we study the bifurcation diagram of the family $QsnSN$ which is the set of all quadratic systems which have at least one finite semi-elemental saddle-node and one infinite semi-elemental saddle-node formed by the collision of two infinite singular points. We study this family with respect to a specific normal form which puts the finite saddle-node at the origin and fixes its eigenvectors on the axes. Our aim is to make a global study of the family $\overline{QsnSN}$ which is the closure of the set of representatives of $QsnSN$ in the parameter space of that specific normal form. This family can be divided into three different subfamilies according to the position of the infinite saddle-node, namely: (A) with the infinite saddle-node in the horizontal axis, (B) with the infinite saddle-node in the vertical axis and (C) with the infinite saddle-node in the bisector of the first and third quadrants. These three subfamilies modulo the action of the affine group and times homotheties are four-dimensional. Here, we provide the complete study of the geometry with respect to a normal form of the first two families, (A) and (B). The bifurcation diagram for the subfamily (A) yields 38 phase portraits for systems in $\overline{QsnSN(A)}$ (29 in $QsnSN(A)$) out of which only three have limit cycles and 13 possess graphics. The bifurcation diagram for the subfamily (B) yields 25 phase portraits for systems in $\overline{QsnSN(B)}$ (16 in $QsnSN(B)$) out of which 11 possess graphics. None of the 25 portraits has limit cycles. Case (C) will yield many more phase portraits and will be written separately in a forthcoming new paper. Algebraic invariants are used to construct the bifurcation set. The phase portraits are represented on the Poincaré disk. The bifurcation set of $\overline{QsnSN(A)}$ is the union of algebraic surfaces and one surface whose presence was detected numerically. All points in this surface correspond to connections of separatrices. The bifurcation set of $\overline{QsnSN(B)}$ is formed only by algebraic surfaces.

Keywords: Quadratic differential systems; finite saddle-node; infinite saddle-node; phase portraits; bifurcation diagram; algebraic invariants.