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GLOBAL PHASE PORTRAITS OF QUADRATIC POLYNOMIAL DIFFERENTIAL SYSTEMS WITH A SEMI-ELEMENTAL TRIPLE NODE

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Planar quadratic differential systems occur in many areas of applied mathematics. Although more than one thousand papers have been written on these systems, a complete understanding of this family is still missing. Classical problems, and in particular, Hilbert's 16th problem [Hilbert, 1900, 1902], are still open for this family. In this article, we make a global study of the family QT$\overline{\text{N}}$ of all real quadratic polynomial differential systems which have a semi-elemental triple node (triple node with exactly one zero eigenvalue). This family modulo the action of the affine group and time homotheties is three-dimensional and we give its bifurcation diagram with respect to a normal form, in the three-dimensional real space of the parameters of this form. This bifurcation diagram yields 28 phase portraits for systems in QT$\overline{\text{N}}$ counting phase portraits with and without limit cycles. Algebraic invariants are used to construct the bifurcation set. The phase portraits are represented on the Poincaré disk. The bifurcation set is not only algebraic due to the presence of a surface found numerically. All points in this surface correspond to connections of separatrices.

Keywords: Quadratic differential systems; semi-elemental triple node; phase portraits; bifurcation diagram; algebraic invariants.

1. Introduction, Brief Review of the Literature and Statement of Results

In this paper, we call *quadratic differential systems* or simply *quadratic systems*, differential systems of the form

$$\dot{x} = p(x, y), \quad \dot{y} = q(x, y), \tag{1}$$

where p and q are polynomials over $\mathbb{R}$ in x and y such that the $\max(\deg(p), \deg(q)) = 2$. To such a system one can always associate the quadratic vector field

$$X = p\frac{\partial}{\partial x} + q\frac{\partial}{\partial y}, \tag{2}$$

as well as the differential equation

$$qdx - pdy = 0. \tag{3}$$

The class of all quadratic differential systems (or quadratic vector fields) will be denoted by QS.

We can also write system (1) as

$$\begin{aligned} \dot{x} &= p_0 + p_1(x, y) + p_2(x, y) = p(x, y), \\ \dot{y} &= q_0 + q_1(x, y) + q_2(x, y) = q(x, y), \end{aligned} \tag{4}$$