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KNEADING THEORY AND BIFURCATIONS FOR A CLASS OF DEGREE ONE CIRCLE MAPS: THE ARNOL'D TONGUES REVISITED

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ABSTRACT. In this paper we introduce the kneading theory developped by Alsedà and Mañosas in [3] to describe bifurcations for a generic family of bimodal degree one circle maps and preserving orientation circle homeomorphisms.

1. INTRODUCTION

The map $g_w(x) = x + w$ can be seen as the superposition of two simple sinusoidal oscillators where $x \in \mathbb{S}^1$ represents the value of the phase of one of the oscillators after the other has done one oscillation. The term $w \in [0, 1)$ represents the ratio of the frequencies of the two oscillators.

When w is an irrational number the motion of the systems is called *quasiperiodic* and if w is a rational number then the motion is called *periodic*.

The more general map

$$H_{b,w}(x) = x + w + \frac{b}{2\pi}\sin(2\pi x) \tag{1}$$

where $x \in \mathbb{R}$ and $(b, w) \in \mathbb{R}^+ \times \mathbb{R}$ was introduced by Arnol'd [4] to study the behaviour of the motion of the system when a non-linear term is added. The resultant family of maps has been used to study some variety of forced systems where there are two competing frecuencies, for example, the case of a sinusoidally driven pendulum.

Depending on the range on b, the family of maps have different behaviours which has been consider in the literature (see [4], [11], [6], [15] and [9]).

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