

On ω -limit sets and related topics for triangular maps on the unit square.

Francisco Balibrea Gallego and Juan Luis García Guirao*

30th April 2003

Abstract

The main aim of the present paper is to make a revision about the situation of some classical problems concerning ω -limit sets for the family of two-dimensional *triangulars* maps on the unit square I^2 , $I = [0, 1]$. Results on the characterization, description, universality and Hausdorff-closure properties of this kind of sets are presented jointly with some open problems.

1 Introduction and Notation.

Given a discrete dynamical system (X, ϕ) where X is a compact metric space and ϕ is a continuous map from X into itself ($\phi \in \mathcal{C}(X)$), one important problem to understand the dynamics of the system is to know the asymptotic behaviour of the *orbits* of all points in X ($Orb_\phi(x) = (\phi^n(x))_{n=0}^\infty$ where $\phi^n(x) = \phi(\phi^{n-1}(x))$ for $n \geq 1$ and ϕ^0 is the identity on X) given by study of their ω -limit sets. Recall that a point y belongs to the ω -*limit set* of x by ϕ , denoted by $\omega_\phi(x)$, if there exists a sequence of positive integers $(n_k)_{k=0}^\infty$ such that $\phi^{n_k}(x) \rightarrow y$ where $n_k \rightarrow \infty$.

In this general setting some problems can be stated:

- (P1) Obtain a topological characterization of the ω -limit sets for the elements of a class $\mathcal{F} \subseteq \mathcal{C}(X)$. This family of sets can be denoted by $\mathcal{W}_\mathcal{F}$.

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This paper has been partially supported by MCYT (Ministerio de Ciencia y Tecnología, Spain) and FEDER (Fondo Europeo de Desarrollo Regional), grants BEC2001-0535 and BFM2002-03512; Fundación Séneca (Comunidad Autónoma de la Región de Murcia), grant PI-8-00807-FS-01 and JCCM (Junta de Comunidades de Castilla-La Mancha), grant PAC-02-002.

Mathematics Subject

Classification 2000: Primary 37B20, Secondary 37B99.

Keywords and phrases:

Discrete dynamical system, triangular map and ω -limit set.