

Continua with empty interior as ω -limit sets

FRANCISCO BALIBREA AND JUAN LUIS GARCÍA-GUIRAO*

ABSTRACT. Continua in \mathbb{R}^n with empty interior form a wide class of continua which can be obtained as ω -limit sets. As examples of them, Peano continua (locally connected continua) are *orbit enclosing* (the point generating the set belongs to it).

In this paper we are dealing mainly with *dendrites* which are Peano continua not containing closed simple curves. Although they appear in several setting, we are concentrating in two of them; dendrites as Julia sets on complex dynamics and dendrites in connection with the problem of constructing continuous and chaotic (in Devaney's sense) maps (there is a point with dense orbit and the set of periodic points is dense in the space) with positive topological entropy. We prove that there exists a planar dendrite (Wazewski's dendrite) which is ω -universal. It means that it is possible to get an universal sequence S generating such dendrite and such that every dendrite in \mathbb{R}^n for every n can be generated taking subsequences of S . Additionally, associated to each finite set of points in the real unit interval $[0, 1] = I$ we construct a dendrite and define a continuous map on it chaotic in the Devaney's sense and with positive topological entropy.

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1. PRELIMINARY RESULTS

Let \mathbb{K} be a compact and connected metric space. We call it a *continuum space* or simply a *continuum*.

Of special interest for their geometric and topological structure are the continua $\mathbb{K} \subset \mathbb{R}^n$ and in particular, when $n = 2$, the *plane continua*.

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