DESCRIPTION OF ω-LIMIT SETS OF A TRIANGULAR MAP ON I^2.

FRANCISCO BALIBREA[1] , J.L. GARCÍA GUIRAO[2] AND J.I. MUÑOZ CASADO[3].

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ABSTRACT. In this paper we describe the ω-limit sets of all points in I^2 for a triangular map given by Kolyada. We prove for such a map the following uniform property:

All points of I^2 except those of the fiber $I_0 = \{0\} \times [0,1]$, which are fixed points, have as infinity ω-limit set the fiber I_0 .

1. INTRODUCTION

Let $I^2 = [0,1] \times [0,1]$ be the bidimensional unit square. We say that a continuous map $F : I^2 \to I^2$ is a *triangular map* when it is of the form

$$F(x,y) = (f(x), g(x,y))$$

where $f : I \to I$, $g : I^2 \to I$ are continuous and $I = [0,1]$. If $F : I^2 \to I^2$ is a triangular map, F^0 refers to the identity map, F^1 is F, and for each integer $n \geq 0$ the $(n+1)th$ *iterate*, F^n, is the map F composed with itself n times. A point $(a,b) \in I^2$ is a *periodic point* when $F^n(a,b) = (a,b)$ for some $n \in \mathbb{N}$ (the smallest of these values n is called the *order* or *period* of the periodic point). When $n = 1$ we say that $(a,b) \in I^2$ is a *fixed point*. Let $(a,b) \in I^2$, the ω-*limit set* of (a,b) by F, denoted by $\omega_F(a,b)$, is the set of limit points of the sequence $(F^n(a,b))_{n=0}^{\infty}$

An interesting problem is to study the topological dynamics of these maps and establish the significant differences with the dynamics of continuous interval maps. To this end, S. Kolyada has introduced a triangular map where the set of periodic points coincides with the set of fixed points and with the property that the ω-limit set of the point $(1,1)$ is the fiber $I_0 = \{0\} \times I$. Besides, for such a map the fiber I_0 is composed of the set of its fixed points.

In this paper we are proving that the asymptotic behavior of $(1,1)$ is a common property for all points in $I^2 \backslash I_0$, that is, every point in $I^2 \backslash I_0$ has the set I_0 as its ω-limit set. This uniform property, on one hand, reinforces the differences in behavior between the class of triangular maps and interval maps because, in the last case a map with only fixed points can not have infinity ω-limit sets.

1. *Departamento de Matemáticas. Universidad de Murcia, 30100-Murcia, Spain;*

2. *Departamento de Fundamentos del Análisis Económico. Universidad de Alicante, 03071-Alicante, Spain;*

3. *Departamento de Matemáticas. I.E.S. J. Ibáñez Martín, 30800-Lorca(Murcia), Spain.*
E-mail address: *balibrea@fcu.um.es, jlgg@merlin.fae.ua.es and cardanio@.eresmas.com*