Journal of Difference Equations and Applications, 2003 Vol. 9 (3/4), pp. 289–304



## On $\omega$ -limit Sets of Triangular Maps on the Unit Cube

F. BALIBREA GALLEGO<sup>a,\*</sup>, J.L. GARCÍA GUIRAO<sup>b,†</sup> and J.I. MUÑOZ CASADO<sup>c,‡</sup>

<sup>a</sup>Departamento de Matemáticas, Universidad de Murcia, 30100-Murcia, Spain; <sup>b</sup>Departamento de Fundamentos del Análisis Económico, Universidad de Alicante, 03071-Alicante, Spain; <sup>c</sup>Departamento de Matemáticas, I.E.S. J. Ibáñez Martín, 30800-Lorca (Murcia), Spain

(Received 29 March 2002; In final form 21 August 2002)

Dedicated to Professor Alexander N. Sharkovsky on the occasion of his 65th Birthday.

In this paper, we construct a triangular map F on  $I^3$  (I = [0, 1]) holding the following uniform property: All points  $(a, b, c) \in I^3$  except those of the face  $I_0^2 = \{0\} \times I^2$ , which are fixed by F, have as  $\omega$ -limit set the face  $I_0^2$ . So, we are able to describe the family  $\mathscr{W}(F) = \{\omega_F(a, b, c) : (a, b, c) \in I^3\}$  for a continuous endomorphism defined in a compact metric space of dimension higher than two, establishing that  $\mathscr{W}(F) = I_0^2 \cup \{(0, b, c)\}_{(b, c) \in I^2}$ .

Keywords: Discrete dynamical system; Triangular map; ω-Limit set; Unit cube

2002 Mathematics Subject Classification: 37E99; 37B99

## INTRODUCTION AND NOTATION

Given a discrete dynamical system  $(X, \phi)$  where X is a compact metric space and  $\phi$  is a continuous map from X into itself ( $\phi \in C(X, X)$ ), to understand the dynamics of the system is to know the asymptotic behavior of the *orbits* of all points in X ( $\operatorname{Orb}_{\phi}(x) = \{\phi^n(x)\}_{n=0}^{\infty}$  where  $\phi^n(x) = \phi(\phi^{n-1}(x))$  for  $n \ge 1$  and  $\phi^0$  is the identity on X) given by the study of their  $\omega$ -limit sets. Recall that a point y belongs to the  $\omega$ -limit set of x by  $\phi$ , denoted by  $\omega_{\phi}(x)$ , if there exists an increasing sequence of positive integers  $\{n_k\}_{k=0}^{\infty}$  such that  $\phi^{n_k}(x) \to y$  where  $k \to \infty$ .

In this general setting some problems can be stated:

(P1) Obtain the topological characterization of the  $\omega$ -limit sets of all points for some class  $\mathscr{F} \subset C(X, X)$ . This family of sets can be denoted by  $W_{\mathscr{F}}$ .

(P2) Given  $A \in \mathscr{W}_{\mathscr{F}}$  construct a continuous map  $\psi \in \mathscr{F}$  with a point  $x \in X$  such that  $\omega_{\psi}(x) = A$ .

<sup>\*</sup>Corresponding author. E-mail: balibrea@um.es

<sup>&</sup>lt;sup>†</sup>E-mail: jlgg@merlin.fae.ua.es

<sup>\*</sup>E-mail: cardanio@eresmas.com

ISSN 1023-6198 print/ISSN 1563-5120 online @ 2003 Taylor & Francis Ltd DOI: 10.1080/1023619021000047734