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Dynamics of a Lotka–Volterra map

by

Francisco Balibrea (Murcia), **Juan Luis García Guirao** (Cartagena), **Marek Lampart** (Opava) and **Jaume Llibre** (Barcelona)

Abstract. Given the plane triangle with vertices $(0,0)$, $(0,4)$ and $(4,0)$ and the transformation $F:(x,y)\mapsto(x(4-x-y),xy)$ introduced by A. N. Sharkovskiĭ, we prove the existence of the following objects: a unique invariant curve of spiral type, a periodic trajectory of period 4 (given explicitly) and a periodic trajectory of period 5 (described approximately). Also, we give a decomposition of the triangle which helps to understand the global dynamics of this discrete system which is linked with the behavior of the Schrödinger equation.

1. Introduction and statement of the main results. Two-dimensional continuous transformations of the plane, $G:(x,y)\mapsto(f(x,y),g(x,y))$, have been considered for a long time to describe many phenomena coming from population dynamics, economy theory, social sciences and engineering.

In most cases there exist compact subsets $X\subset\mathbb{R}^2$, invariant under the action of the transformation (i.e., $G(X)\subseteq X$), where the most interesting part of the dynamics of the system is developed. If we see them as two-dimensional discrete dynamical systems, i.e. couples of the form $(X,G|_X)$, the interest is focused on the behavior of points of X, i.e., how the trajectories of all points evolve under the action of G.

In applications, the maps f and g are usually piecewise polynomial on X, i.e., there exists a finite partition of X, $\{X_i\}_{i=1}^n$, such that f,g restricted to

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