AN ENVIRONMENT FOR COMPUTING TOPOLOGICAL ENTROPY FOR SKEW-PRODUCT TRANSFORMATIONS

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The main objective of this paper is to introduce a specific software package for computing topological entropy of two-dimensional skew-product transformations. The Kneading Theory establishes the necessary background for this study is some particular cases. This is not only a hard but also a computationally expensive problem. The CAS *Mathematica* provides a suitable environment for implementing the algorithm associated to the rigorous computation of the topological entropy of the considered systems.

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Topics: Computational Sciences.

Nonlinear dynamic systems models are ubiquitous throughout the sciences and engineering. Efficient computational analysis of specific dynamical systems is often a critical component for the successful completion of a research or design project.

We live in a dynamical world. For many reasons, we want to understand these dynamics: to predict the weather, to prevent heart attacks and limit spread of infections diseases, to control agricultural pests, to farsee the consequences on man´s activities on the global climate and the impact of climate changes that might result from these activities, to design both more reliable and more efficient machines, and so on.

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Questions about how processes evolve and change in time are really important and of broad usefulness.

Indeed in various domains of science many situations can be, at least approximately, modelled in a very simple way via difference equations of the form

$$x_{n+1} = f(x_n), \quad n = 0, 1, 2, \ldots$$

where f is a particular function which define a dynamical system. The notion of dynamical system is the mathematical formalization of the more general concept of a deterministic process. The future state of many Physical, Chemical, Biological, Engineering, Economical and even Social Systems can be predicted, to a certain extent, by knowing the present state, x_0, and the law governing its evolution, f.

Because of that, this work could be a useful object for researchers in those areas who use dynamical systems as model tools in their studies.

We are not concerned with the problem of how to find such f. Generally, an experimental function f is approximated by an explicitly defined function depending on parameters that are subsequently determined by means of statistical methods.

These dynamical systems are defined by rules of transformations which are needed in order to determine how points in a state space evolve when time elapses. Time can either be discrete or continuous. The traces of points as they move in discrete or continuous time are called trajectories. Dynamical systems theory seeks a comprehensive description of the geometric structures arising from these trajectories.

In most of cases such trajectories are difficult to describe, even is not possible, when it occurs for many of them, the system behaves in a very complicated way.

The scenario:

Let $I = [0,1]$ be the compact unit interval of the real line. We consider *skew-product transformations* on the unit square, that is, surjective continuous maps from I^2 into itself of the form $F : (x, y) \to (f(x), g(x, y))$ ($F \in C_\Delta(I^2)$). In this setting, the maps f and g are respectively called the *basis* and the *fiber* map of F. For every $x \in I$, the maps g_x defined by $g_x(y) = g(x, y)$ form a system of one-dimensional mappings depending continuously on x.

More details on this kind of maps can be found in [16,17,5,6].

These types of systems have a lot of applications in pure mathematics (e.g. in the study of geodesic flows on Riemannian surfaces of constant