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On central configurations of the κn-body problem

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ABSTRACT

We consider planar central configurations of the Newtonian κn-body problem consisting in κ groups of regular n-gons of equal masses, called (κ, n)-crown. We derive the equations of central configurations for a general (κ, n)-crown. When $\kappa = 2$ we prove the existence of a twisted $(2, n)$-crown for any value of the mass ratio. Moreover, for $n = 3, 4$ and any value of the mass ratio, we give the exact number of twisted $(2, n)$-crowns, and describe their location. Finally, we conjecture that for any value of the mass ratio there exist exactly three $(2, n)$-crowns for $n \geq 5$.

1. Introduction

In the N-body problem a configuration is *central* if the acceleration vector for each body is a common scalar multiple of its position vector with respect to the center of mass. The study of central configurations allows to obtain explicit solutions of the N-body problem where the shape remains constant up to rescaling and rotation. While much is known about specific cases, usually involving symmetry or assuming that some bodies are infinitesimally small, less is known about the general structure of the set of central configurations. See Saari [10] for an introduction to the subject.

There are numerous results in the literature demonstrating the existence of central configurations with specific symmetries. See for instance [8] and the references therein. In Montaldi [8] the author prove, using variational arguments, that there exist central configurations for every possible symmetry type, and for any symmetric choice of masses.

We focus on central configurations of the planar N-body problem for $N = \kappa n$, consisting in κ groups of n bodies located at the vertices of regular n-gons. In principle, no conditions on the masses of the same n-gon are imposed. Nevertheless, in the case of two regular n-gons, Zhang and Zhou [15] prove that the masses within each group must be equal. Although it is not known if that condition is necessary for more than two

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