SIAM J. APPLIED DYNAMICAL SYSTEMS
Vol. 9, No. 2, pp. 634–658

A Limit Case of the "Ring Problem": The Planar Circular Restricted $1 + n$ Body Problem*

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Abstract. We study the dynamics of an extremely idealized model of a planetary ring. In particular, we study the motion of an infinitesimal particle moving under the gravitational influence of a large central body and a regular n-gon of smaller bodies as n tends to infinity. Our goal is to gain insight into the structure of thin, isolated rings.

Key words. celestial mechanics, N-body problem, KAM theorem, planetary rings

AMS subject classifications. 70F07, 70F15, 70F45, 70H08

DOI. 10.1137/090771582

1. Introduction.

The study of the dynamics of planetary rings has a long and interesting history which is still under development. An inclusive bibliography of the study of rings is beyond the scope of this paper, but a flavor of the diversity of ideas and techniques employed can be found, for example, in the books [13] and [4] and the references therein. In particular, the influence of interactions with moons orbiting the planet both close to the ring ("shepherds") and far from the ring (via resonance effects) is extremely important.

Our goal in this paper is to study the intrinsic dynamics of a planetary ring. In particular, we create and study a model of a thin ring about a large planet, unaffected by any other bodies, with motion governed by Newtonian gravitation only. Surprisingly, our model has interesting internal dynamics and points out natural relationships between the stability of the motion of ring particles, the width of the ring, and the ratio of the masses of the ring and the planet.

The starting point for our model dates back, at least, to James Clerk Maxwell. In his prize-winning paper of 1890 [10], Maxwell studied several models for Saturn's rings. Under Newtonian, inverse square, gravitational attraction, Maxwell showed that liquid and solid rings would be dynamically unstable. On the other hand, he showed that a ring made up of n small, equal mass bodies arranged in a regular n-gon about the planet and moving in a circular orbit would be stable (at least when $n > 7$ and the planet is large enough; see Moeckel [12]). Such a solution of the $1 + n$ body problem is an example of a "relative equilibrium" orbit

*Received by the editors September 18, 2009; accepted for publication (in revised form) by J. Meiss March 30, 2010; published electronically June 9, 2010.
http://www.siam.org/journals/siads/9-2/77158.html

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