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J_2 Effect and Elliptic Inclined Periodic Orbits in the Collision Restricted Three-Body Problem*

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Abstract. The existence of a new class of inclined periodic orbits of the collision restricted three-body problem is shown. The symmetric periodic solutions found are perturbations of elliptic Kepler orbits, and they exist only for special values of the inclination and are related to the motion of a satellite around an oblate planet.

Key words. collision restricted three-body problem, periodic orbits, symmetric orbits, critical inclination, continuation method

AMS subject classifications. 70F07, 70F15, 70H09, 70H12, 70M20

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1. Introduction.

The launch of Sputnik in October 1957 opened the space age. The use of circular, elliptic, and synchronous orbits, combined with dynamical effects due to the Earth's equatorial bulge, gives rise to an array of orbits with specific properties to support various mission constraints. One example is the Molniya orbit, a highly elliptic 12-hour-period orbit the former USSR originally designed to observe the northern hemisphere. The orbital plane makes an angle of about 63 degrees with the equatorial plane of the Earth, and this is the only value that prevents the orbit itself from rotating slowly within its plane and around the focus.

In what follows we will introduce briefly a few common notions of orbital dynamics, together with the current terminology (sometimes a few centuries old), and state the aim of the paper.

The position of a body on a Keplerian elliptic orbit can be completely characterized by six parameters. One such set of parameters are the classical orbital elements. As the orbital plane is fixed in any inertial frame and passes through the origin, one should first give the position of this plane. In a Cartesian frame with axes xyz, this is given by the inclination i with respect to the xy-plane and the angle Ω from the positive x-axis to the intersection of the orbital plane with the xy-plane. In the classical terminology of astronomy this line is

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