Physica D 239 (2010) 214–219

Contents lists available at ScienceDirect

Physica D

journal homepage: www.elsevier.com/locate/physd

Highly eccentric hip–hop solutions of the 2N–body problem

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ARTICLE INFO

Article history:
Received 26 March 2009
Received in revised form
27 October 2009
Accepted 31 October 2009
Available online 10 November 2009
Communicated by G. Stepan

Keywords:
Spatial 2N body problem
Hip–hop solutions
Topological method

ABSTRACT

We show the existence of families of hip–hop solutions in the equal–mass 2N–body problem which are close to highly eccentric planar elliptic homographic motions of 2N bodies plus small perpendicular non–harmonic oscillations. By introducing a parameter ϵ, the homographic motion and the small amplitude oscillations can be uncoupled into a purely Keplerian homographic motion of fixed period and a vertical oscillation described by a Hill type equation. Small changes in the eccentricity induce large variations in the period of the perpendicular oscillation and give rise, via a Bolzano argument, to resonant periodic solutions of the uncoupled system in a rotating frame. For small $\epsilon \neq 0$, the topological transversality persists and Brouwer's fixed point theorem shows the existence of this kind of solutions in the full system.

1. Introduction

Hip–hop solutions of the equal–mass 2N–body problem are periodic solutions in which all the bodies move in such a way that their positions in configuration space are at the vertices of a regular antiprism for all time.

A regular antiprism is a polyhedron formed by two congruent regular N-gons perpendicular to the line joining their centers and such that their orthogonal projections along this line form a regular 2N-gon, i.e. one of the N-gons has been rotated an angle π/N on its own plane. The polyhedron is completed by connecting both N-gons, which we call the *bases*, by an alternating band of isosceles triangles. The symmetries of the equations of motion when all the masses are equal ensure that if at a given time t_0 the 2N bodies are on the vertices of a regular antiprism and the velocities satisfy the appropriate conditions of symmetry, then they will stay forever on the vertices of an antiprism.

If we take the line joining the centers of the bases to be the z-axis and the center of mass at the origin, then the picture of a hip–hop solution is similar to having two equal planar homographic elliptic solutions on parallel planes, each one rotated through half a central angle with respect to the other, together with an oscillatory motion of the planes along their common perpendicular. The planes will coincide at a given time with opposite velocities, separate in opposite directions, reach a maximum distance and fall again to coincide. The orthogonal projection of both N-gons on the $z = 0$ plane will always be a regular rotating 2N-gon of variable size. A hip–hop solution is a periodic solution of this type, where periodic has the usual meaning of periodic in an ad–hoc rotating reference frame (see [1,2]).

A number of results on hip–hop solutions has been obtained by means of variational methods. With these methods it is possible to find solutions that do not depend on a small parameter (see [3,2] and the references therein for more details). In [1], the authors show that Poincare's argument of analytic continuation can be used to add vertical oscillations to the circular motion of 2N bodies of equal mass occupying the vertices of a regular 2N-gon, and prove the existence of families of hip–hop solutions with eccentricity close to zero. An infinite number of these orbits are 3D choreographies, i.e. all the bodies move on the same non–planar curve at equally spaced time intervals. Symmetric periodic trajectories for the N-body problem (including hip–hop solutions and possible generalizations) have been addressed in full generality by means of variational methods in [4,5]. We mention finally a recent paper [6], in which the authors consider a particular 2N body problem in space with equal masses, where at each instant the bodies form an orbit of the dihedral group D_l with D_l the group of order 2l generated by two rotations of angle π around two secant lines in space meeting at an angle of π/l.

In this work we prove the existence of hip–hop solutions for values of the eccentricity close to 1. Roughly speaking, these solutions are obtained by introducing a small parameter ϵ in order to rescale

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doi:10.1016/j.physd.2009.10.019