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Univalent Baker domains

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Abstract
We classify Baker domains U for entire maps with $f|_U$ univalent into three different types, giving several criteria which characterize them. Some new examples of such domains are presented, including a domain with disconnected boundary in $\mathbb{C}$ and a domain which spirals towards infinity.

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1. Introduction

Let $f : \mathbb{C} \to \mathbb{C}$ be an entire transcendental map. Then f induces a partition of the complex plane into two completely invariant sets: the *Fatou set* and the *Julia set*. The first one, $F(f)$, is defined as the set of points $z \in \mathbb{C}$ for which the sequence of iterates $\{f^n\}_{n\geqslant 0}$ forms a normal family in some neighbourhood of z. Its complement is the Julia set, $J(f)$. Clearly, the Fatou set is an open set of $\mathbb{C}$ while the Julia set is closed. It is a special property of entire transcendental maps that both sets are unbounded. Refer, for example, to [Ber2, BR] for the general description of the dynamics of these maps.

Since $F(f)$ is completely invariant, its connected components must map among themselves. We say that a connected component U of $F(f)$ is *periodic* of period $p \geqslant 1$, if $f^p(U) \subset U$. Note that unlike the case of rational maps, it is possible to have $f^p(U) \neq U$ (see, e.g., [Ber2]).

If U is a periodic component of $F(f)$ of period $p \geqslant 1$, there are only four possible cases:

(a) U is a component of the *attracting basin* of an attracting periodic point $z_0 \in U$ and $f^{np}(z) \underset{n\to\infty}{\longrightarrow} z_0$ for all $z \in U$,
(b) U is a component of the *parabolic basin* of a parabolic point $z_0 \in \partial U$ and $f^{np}(z) \underset{n\to\infty}{\longrightarrow} z_0$ for all $z \in U$,
(c) U is a *Siegel disc*, i.e. U is conformally equivalent to a disc and $f^p|_U$ is analytically conjugate to a rigid rotation,
(d) $f^{np}(z) \underset{n\to\infty}{\longrightarrow} \infty$ for all $z \in U$. In this case, U is called a *Baker domain*.