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ACCESSES TO INFINITY FROM FATOU COMPONENTS

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Abstract. We study the boundary behaviour of a meromorphic map $f : \mathbb{C} \to \widehat{\mathbb{C}}$ on its simply connected invariant Fatou component U. To this aim, we develop the theory of accesses to boundary points of U and their relation to the dynamics of f. In particular, we establish a correspondence between invariant accesses from U to infinity or weakly repelling fixed points of f and boundary fixed points of the associated inner function on the unit disc. We apply our results to describe the accesses to infinity from invariant Fatou components of the Newton maps.

1. Introduction

We consider dynamical systems generated by the iteration of meromorphic maps $f : \mathbb{C} \to \widehat{\mathbb{C}}$. We are especially interested in the case when f is transcendental or, equivalently, when the point at infinity is an essential singularity of f and hence the map is non-rational. There is a natural dynamical partition of the complex sphere $\widehat{\mathbb{C}}$ into two completely invariant sets: the *Fatou set* $\mathcal{F}(f)$, consisting of points for which the iterates $\{f^n\}_{n\geq 0}$ are defined and form a normal family in some neighbourhood, and its complement, the *Julia set* $\mathcal{J}(f)$, where chaotic dynamics occurs. Note that in the transcendental case, we always have $\infty \in \mathcal{J}(f)$. The Fatou set is open and it is divided into connected components called *Fatou components*, which map among themselves. Periodic Fatou components (i.e. components U with $f^p(U) \subset U$ for some period $p \geq 1$) are classified into *basins of attraction* of attracting or parabolic cycles, *rotation domains* (Siegel discs or Herman rings, depending on their genus, where the dynamics behaves like an irrational rotation), or *Baker domains* (components for which f^{pn} converge to infinity as $n \to \infty$ uniformly on compact sets). An *invariant* Fatou component is a component U satisfying $f(U) \subset U$. Components which are not eventually periodic are called *wandering*, and they may or may not converge to infinity under iteration.

In this paper we are interested in the interplay between the dynamics of f on a simply connected invariant Fatou component U, the geometry of the boundary of U and the boundary behaviour of a Riemann map $\varphi : \mathbb{D} \to U$ from the unit disc $\mathbb{D}$ onto U. The main motivation is to understand the structure of the Julia set near infinity of a *Newton map*, i.e. a non-constant non-Möbius meromorphic map

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