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Connectivity of Julia sets of Newton maps: a unified approach

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Abstract. In this paper we present a unified proof of the fact that the Julia set of Newton's method applied to a holomorphic function on the complex plane (a polynomial of degree larger than 1 or a transcendental entire function) is connected. The result was recently completed by the authors' previous work, as a consequence of a more general theorem whose proof spreads among many papers, which consider separately a number of particular cases for rational and transcendental maps, and use a variety of techniques. In this note we present a unified, direct and reasonably self-contained proof which works in all situations alike.

1. Introduction

Newton's method is one of the oldest and best known root-finding algorithms. It is also a motivation which inspired the modern approach to holomorphic dynamics, when the local study turned out to be insufficient for a good understanding of the method applied to complex polynomials.

The global dynamics of Newton's method applied to complex quadratic polynomials is always conjugate to the dynamics of $z \mapsto z^2$, as was already noticed in the early works of E. Schröder and A. Cayley [11], [12], [13], [28], [27]. They also observed that this trivial situation is no longer true when Newton's method is applied to higher degree polynomials, where the boundaries between different basins of attracting fixed points (known nowadays as the Julia set) have, in general, rich and intricate topology.

A good understanding of the topology of the Julia set of Newton's method applied to polynomials or transcendental entire functions is interesting not only from the point of view of holomorphic dynamics but has also concrete numerical applications, see e.g. [20]. One of the questions which has attracted much attention

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