THE MINIMUM TREE FOR A GIVEN ZERO-ENTROPY PERIOD

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We answer the following question: given any $n \in \mathbb{N}$, which is the minimum number of endpoints e_n of a tree admitting a zero-entropy map f with a periodic orbit of period n? We prove that $e_n = s_1 s_2 \cdots s_k - \sum_{i=2}^{k} s_i s_{i+1} \cdots s_k$, where $n = s_1 s_2 \cdots s_k$ is the decomposition of n into a product of primes such that $s_i \leq s_{i+1}$ for $1 \leq i < k$. As a corollary, we get a criterion to decide whether a map f defined on a tree with e endpoints has positive entropy: if f has a periodic orbit of period m with $e_m > e$, then the topological entropy of f is positive.

1. Introduction

In the last decades, many authors have studied the dynamical behaviour of continuous self-maps of one-dimensional spaces. There are also some books where most of the results are collected in (see, e.g., [3, 7]). In particular, the study of the set of periods of continuous maps $f : X \to X$, where X is a tree (a graph without circles), has been one of the problems that have centered the attention. The first and most famous result in this direction is Šarkovs'kiĭ's theorem [13], which gives a complete characterization of the set of periods of f when X is a closed interval of the real line. Baldwin [5] extended this result to the case of an n-star (a tree consisting of n edges attached to a unique central point). Recently, Alsedà, Juher, and Mumbrú [2] have developed a characterization of the set of periods of f when X is any generic tree, in terms of the topological structure of X (number and arrangement of vertices, edges, and endpoints of X).

One way to study the dynamical complexity of a continuous map $f : X \to X$ of a compact metric space is computing its topological entropy, a nonnegative constant which measures how the iterates of the map mix the points of X (see [1]). It is known that an interval or line map with positive topological entropy is *chaotic* in the sense of Li and Yorke (see [11]). If X is a general compact metric space, the same result has been recently obtained in [6]. It is also well known that the topological entropy of f is closely related with the sizes of the periodic orbits exhibited by f. Some results give upper or lower bounds of the set of periods of f depending on whether f has a positive entropy or not. Of course, these bounds depend strongly on the particular space X under consideration.

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