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The positive entropy kernel for some families of trees

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Abstract

The fact that a continuous self-map of a tree has positive topological entropy is related to the number of different greatest odd divisors (gods) exhibited by its set of periods. Llibre and Misiurewicz (1993 *Topology* **32** 649–64) and Blokh (1992 *Nonlinearity* **5** 1375–82) give generic upper bounds for the maximum number of gods that a zero entropy tree map $f: T \longrightarrow T$ can exhibit, in terms of the number of endpoints and edges of T. In this paper we compute exactly the minimum of the positive integers n such that the entropy of each tree map $f: T \longrightarrow T$ exhibiting more than n gods is necessarily positive, for the family of trees which have a subinterval containing all the branching points (this family includes the interval and the stars). We also compute which gods are admissible for such maps.

Mathematics Subject Classification: 37E25, 37B40

1. Introduction

In the framework of discrete dynamical systems, the study of the set of periods for continuous self-maps of one-dimensional spaces has attracted attention in the last few decades. In particular, we will focus on the study of the set of periods of maps $f: T \longrightarrow T$, where T is a tree (a graph without circles). The first and most famous result in this direction is Sharkovsky's theorem [12], which gives a complete characterization of the set of periods of f when T is a closed interval. Later on, a similar characterization was also given for n-stars (trees consisting of n edges attached at a unique central point) by Baldwin (in [6]). Recently, Alsedà *et al* (see [2]) characterized the set of periods of f when T is any generic tree, in terms of the topological structure of T.

One way to study the dynamical complexity of a continuous map $f: X \longrightarrow X$ of a compact metric space is computing its *topological entropy*, a non-negative constant which measures

