IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **43** (2010) 285201 (14pp) doi:10.1088/1751-8113/43/28/285201

On a degenerate Hopf bifurcation

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Received 18 February 2010, in final form 11 May 2010
Published 14 June 2010
Online at stacks.iop.org/JPhysA/43/285201

Abstract
We consider a one-parameter family of $\mathcal{C}^3$ differential equations $\dot{x} = f(x, \varepsilon)$ in $\mathbb{R}^m$ with $m \geqslant 5$ and a parameter ε. We assume that for each ε the differential equation has an equilibrium point $x(\varepsilon)$, that the Jacobian matrix $f_x(x(\varepsilon), \varepsilon)$ has two pairs of complex eigenvalues $\varepsilon\alpha_i \pm \mathrm{i}(\beta + \varepsilon\beta_i) + O(\varepsilon^2)$ for $i = 1, 2$ with $\alpha_1\alpha_2\beta \neq 0$, and that the other eigenvalues are $\varepsilon c_k + O(\varepsilon^2) \in \mathbb{R}$ with $c_k \neq 0$ for $k = 5, \ldots, m$. We note that when $\varepsilon = 0$ the eigenvalues of the Jacobian matrix for the equilibrium point $x(0)$ are $\pm\mathrm{i}\beta$ with multiplicity 2, and 0 with multiplicity $m - 4$. We study the degenerate Hopf bifurcation which takes place in this parameter family at $\varepsilon = 0$.

PACS numbers: 02.30.Hq, 02.30.Oz
Mathematics Subject Classification: 34C23, 34C29, 37G15

1. Introduction

Consider a system of ordinary differential equations

$$\dot{x} = f(x, \varepsilon), \qquad x = (x_1, \ldots, x_m), \tag{1}$$

having an equilibrium point $x(\varepsilon)$ for each ε in a neighborhood U of $\varepsilon = 0$, i.e. $f(x(\varepsilon), \varepsilon) = 0$ for $\varepsilon \in U$. The numbers x_k, for $k = 1, \ldots, m$, are real variables, ε is the bifurcation parameter, and the dot refers to differentiation with respect to the independent variable t.

We can always assume that the equilibrium point $x(0)$ is at the origin of the coordinates, i.e. that $x(0) = 0$. We consider the linear variational equation

$$\dot{x} = Ax, \qquad A = f_x(0, 0), \tag{2}$$

where $f_x(0, 0)$ denotes the Jacobian matrix of the function f evaluated at $(x, \varepsilon) = (0, 0)$. Clearly, every pair of conjugated purely imaginary eigenvalues of A gives rise to the periodic solutions of (2). We are concerned with the classical problem of finding periodic solutions

