Periodic Orbits near Equilibria

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Abstract

Lyapunov, Weinstein, and Moser obtained remarkable theorems giving sufficient conditions for the existence of periodic orbits emanating from an equilibrium point of a differential system with a first integral. Using averaging theory, we establish a similar result for a differential system *without* assuming the existence of a first integral. Our result can also be interpreted as a kind of special Hopf bifurcation. © 2010 Wiley Periodicals, Inc.

1 Introduction

Consider a system of ordinary differential equations

$$\dot{x} = f(x), \quad x = (x_1, \dots, x_m), \tag{1.1}$$

near an equilibrium point that we assume to be the origin $x = 0$. The x_k, for $k = 1, \dots, m$, are real variables, and the dot refers to differentiation with respect to the independent variable t.

A special role in the theory is played by Hamiltonian systems

$$\dot{x}_k = H_{x_{n+k}}, \quad \dot{x}_{n+k} = -H_{x_k}, \quad k = 1, \dots, n, \tag{1.2}$$

where H_{x_l} denotes the partial derivative of the Hamiltonian $H(x_1, \dots, x_{2n})$ with respect to the variable x_l. We can combine equations (1.2) into a single equation by writing

$$\dot{x} = JH_x, \quad J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \tag{1.3}$$

where H_x denotes the gradient of H and where I is the $n \times n$ identity matrix.

For the equilibrium point $x = 0$, we consider the linear variational equation

$$\dot{x} = Ax, \quad A = f_x(0), \tag{1.4}$$

Communications on Pure and Applied Mathematics, Vol. LXIII, 1225–1236 (2010)