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Limit cycles from a four-dimensional centre in $\mathbb{R}^m$ in resonance $p:q$

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Given positive coprime integers p and q, we consider the linear differential centre $\dot{x} = Ax$ in $\mathbb{R}^m$ with eigenvalues $\pm pi$, $\pm qi$ and 0 with multiplicity $m-4$. We perturb this linear centre in the class of all polynomial differential systems of the form linear plus a homogeneous nonlinearity of degree $p+q-1$, i.e. $\dot{x} = Ax + \varepsilon F(x)$, where every component of $F(x)$ is a linear polynomial plus a homogeneous polynomial of degree $p+q-1$. When the displacement function of order ε of the perturbed system is not identically zero, we study the maximal number of limit cycles that can bifurcate from the periodic orbits of the linear differential centre.

Keywords: periodic orbit; averaging theory; limit cycles; resonance $p:q$

AMS Subject Classifications: 34C29; 34C25; 47H11

1. Introduction

In the qualitative theory of polynomial differential systems, the study of their limit cycles and mainly the obtention of information on their number for a given polynomial differential system is one of the main topics. We recall that for a differential system a *limit cycle* is a periodic orbit isolated in the set of all its periodic orbits.

In dimension two, i.e. in the plane, the two main problems related with limit cycles are: first, the study of the number of limit cycles depending on the degree of the polynomial differential system. This is an old problem proposed by D. Hilbert in 1900, known as the 16th Hilbert problem (see the surveys [1,2] for details), and second the study of how many limit cycles emerge from the periodic orbits of a given centre when we perturb it inside a certain class of differential systems [3].

Since the study of limit cycles and mainly the obtention of information on their number for a given polynomial differential system is in general a very difficult problem (almost impossible), there are hundreds of papers trying to solve these questions in the plane for many particular families of polynomial systems, see the references quoted in the book [3] and in the surveys [1,2].

These problems have been studied intensively in dimension two, and unfortunately the results are far from being satisfactory. In fact, the Riemann conjecture and the 16th Hilbert problem are the two unique problems of the famous list of Hilbert which are not solved.

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