J. Math. Anal. Appl. 389 (2012) 754–768

Contents lists available at SciVerse ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

Bifurcation of limit cycles from a 4-dimensional center in $\mathbb{R}^m$ in resonance $1:N$

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ARTICLE INFO

Article history:
Received 3 February 2011
Available online 17 December 2011
Submitted by C.E. Wayne

Keywords:
Periodic orbit
Averaging theory
Limit cycles
Resonance $1:N$

ABSTRACT

For every positive integer $N \geqslant 2$ we consider the linear differential center $\dot{x} = Ax$ in $\mathbb{R}^m$ with eigenvalues $\pm i$, $\pm Ni$ and 0 with multiplicity $m-4$. We perturb this linear center inside the class of all polynomial differential systems of the form linear plus a homogeneous nonlinearity of degree N, i.e. $\dot{x} = Ax + \varepsilon F(x)$ where every component of $F(x)$ is a linear polynomial plus a homogeneous polynomial of degree N. When the displacement function of order ε of the perturbed system is not identically zero, we study the maximal number of limit cycles that can bifurcate from the periodic orbits of the linear differential center. In particular, we give explicit upper bounds for the number of limit cycles.

1. Introduction

In the qualitative theory of polynomial differential systems the study of their limit cycles is one of the main topics. We recall that for a differential system a *limit cycle* is a periodic orbit isolated in the set of all its periodic orbits. Two main questions arise in this setting in dimension two: the study of the number of limit cycles depending on the degree of the polynomial (see [10,11] for details in dimension two), and the study of how many limit cycles emerge from the periodic orbits of a center when we perturb it inside a given class of differential equations (see [8] for details). These problems have been studied intensively in dimension two. Our main aim is to bring this study to higher dimension.

In this paper we study how many limit cycles emerge from the periodic orbits of a center when we perturb it inside a given class of differential equations in dimension higher than two. More precisely given $m \geqslant 5$ we consider the linear differential center

$$\frac{dx}{dt} = \dot{x} = Ax \tag{1}$$

in $\mathbb{R}^m$, where

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0022-247X/$ – see front matter
doi:10.1016/j.jmaa.2011.12.018