

## Dynamical aspects of multi-round horseshoe-shaped homoclinic orbits in the RTBP

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**Abstract** We consider the planar restricted three-body problem and the collinear equilibrium point  $L_3$ , as an example of a center×saddle equilibrium point in a Hamiltonian with two degrees of freedom. We explore numerically the existence of symmetric and non-symmetric homoclinic orbits to  $L_3$ , when varying the mass parameter  $\mu$ . Concerning the symmetric homoclinic orbits (SHO), we study the multi-round,  $m$ -round, SHO for  $m \geq 2$ . More precisely, given a transversal value of  $\mu$  for which there is a 1-round SHO, say  $\mu_1$ , we show that for any  $m \geq 2$ , there are countable sets of values of  $\mu$ , tending to  $\mu_1$ , corresponding to  $m$ -round SHO. Some comments on related analytical results are also made.

**Keywords** Invariant manifolds · Multi-round homoclinic orbits · Restricted three-body problem · Symmetric homoclinic orbits · Homoclinic connection to  $L_3$

### 1 Introduction

It is well known that homoclinic and heteroclinic connections of hyperbolic objects play an important role in the study of dynamical systems from a global point of view. They are also

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