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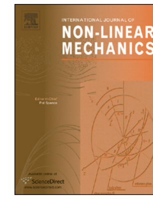

Integrability and limit cycles of the Moon–Rand system

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ABSTRACT

We study the Darboux integrability of the Moon–Rand polynomial differential system. Moreover we study the limit cycles of the perturbed Moon–Rand system bifurcating from the equilibrium point located at the origin, when it is perturbed inside the class of all quadratic polynomial differential systems in $\mathbb{R}^3$, and we prove that at first order in the perturbation parameter ε the perturbed system can exhibit one limit cycle, and that at second order it can exhibit four limit cycles bifurcating from the origin. We provide explicit expressions of these limit cycles up to order $O(\varepsilon^2)$.

1. Introduction and statement of the main results

The Moon–Rand system (introduced by Moon and Rand) was developed to model the control of flexible space structures (see [12,6,7,11]). It is a differential system in $\mathbb{R}^3$ of the form:

$$\dot{x} = y,\quad \dot{y} = -x - xz,\quad \dot{z} = -\lambda z + \sum_{i=0}^{2} c_i x^i y^{2-i}, \tag{1}$$

where $(x,y,z) \in \mathbb{R}^3$ are the variables, $\lambda > 0$, c_i are real parameters and the dot indicates derivative with respect to the time t. In [11] Mahdi et al. studied the Hopf bifurcation of the equilibrium point located at the origin of system (1) using the reduction to the center manifold and studying the Hopf bifurcation on this surface. They found that 2 limit cycles can bifurcate from the origin of system (1). In this paper we study the case when $\lambda = 0$, i.e.

$$\dot{x} = y,\quad \dot{y} = -x - xz,\quad \dot{z} = \alpha x^2 + \beta xy + \gamma y^2, \tag{2}$$

where $(x,y,z) \in \mathbb{R}^3$ are the state variables and (α, β, γ) are real parameters. We emphasize that the center manifold is now $\mathbb{R}^3$.

We note that system (2) is a family of quadratic systems in a three-dimensional space. Quadratic systems in $\mathbb{R}^3$ are the simplest systems after the linear ones. Examples of such systems are the well-known Lorenz system, Rössler system and Rikitake system, among others. These have been investigated in the last decades from different dynamical points of view. Despite their simplicity, quadratic polynomial differential systems are far from being completely understood. In this paper we contribute to the understanding of the dynamics and complexity of system (2) by studying more deeply the existence of limit cycles and its integrability.

The first aim of this paper is to study the existence of Darboux first integrals of system (2), using the Darboux theory of integrability (originated in paper [4]). For the present state of this theory see Chapter 8 of [5], the paper [8], and the references quoted in them. We recall that a *first integral of Darboux type* is a first integral H which is a function of Darboux type (see below (4) for a precise definition).

The vector field associated to system (2) is

$$\mathbf{X} = y\frac{\partial}{\partial x} - x(1+z)\frac{\partial}{\partial y} + (\alpha x^2 + \beta xy + \gamma y^2)\frac{\partial}{\partial z}.$$

Let $U \subset \mathbb{R}^3$ be an open set. We say that a non-constant function $H : U \to \mathbb{R}$ is a *first integral* of the polynomial vector field $\mathbf{X}$ on U if $H(x(t), y(t), z(t))$ is constant for all values of t for which a solution $(x(t), y(t), z(t))$ of $\mathbf{X}$ is defined on U. Clearly H is a first integral of $\mathbf{X}$ on U if and only if

$$\mathbf{X}H = y\frac{\partial H}{\partial x} - x(1+z)\frac{\partial H}{\partial y} + (\alpha x^2 + \beta xy + \gamma y^2)\frac{\partial H}{\partial z} = 0 \tag{3}$$

on U. We say that system (2) is *completely integrable* if there exists two functionally independent first integrals H_1 and H_2. We recall that two first integrals H_1 and H_2 are *functionally independent* if

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