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Dynamics of two Einstein–Friedmann cosmological models

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Abstract We describe completely the dynamics of the two Einstein–Friedmann cosmological models, which can be characterized by the Hamiltonians

$$H = \frac{1}{2}(p_y^2 - p_x^2) + e^{2x} V(y),$$

with the cosmological potentials $V(y) = e^{\lambda y}$, or $V(y) = (a + by)e^y$ with $\lambda ab \neq 0$.

1 Introduction and statement of the main results

The present work is devoted to the Einstein–Friedmann cosmological models, which can be characterized by the Hamiltonian

$$H = \frac{1}{2}(p_y^2 - p_x^2) + e^{2x} V(y), \tag{1}$$

where $V(y) = e^{\lambda y}$ or $V(y) = (a + by)e^y$ with $\lambda ab \neq 0$ are cosmological potentials. For more details on these two special models see subsections 2.2 and 3.1 [10], and for more details on the general Einstein–Friedmann cosmological models see [4,7].

The Hamiltonian system with two degrees of freedom associated with the Hamiltonian

$$H = \frac{1}{2}(p_y^2 - p_x^2) + e^{2x+\lambda y} \tag{2}$$

is

$$
\begin{aligned}
\dot{x} &= \frac{\partial H}{\partial p_x} = -p_x, \\
\dot{y} &= \frac{\partial H}{\partial p_y} = p_y, \\
\dot{p}_x &= -\frac{\partial H}{\partial x} = -2e^{2x+y\lambda}, \\
\dot{p}_y &= -\frac{\partial H}{\partial y} = -\lambda e^{2x+y\lambda}.
\end{aligned}
\tag{3}
$$

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