ON PERIODIC SOLUTIONS OF 2-PERIODIC LYNESS' EQUATIONS

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We study the existence of periodic solutions of the nonautonomous periodic Lyness' recurrence $u_{n+2} = (a_n + u_{n+1})/u_n$, where $\{a_n\}_n$ is a cycle with positive values a, b and with positive initial conditions. It is known that for a = b = 1 all the sequences generated by this recurrence are 5-periodic. We prove that for each pair $(a, b) \neq (1, 1)$ there are infinitely many initial conditions giving rise to periodic sequences, and that the family of recurrences have almost all the even periods. If $a \neq b$, then any odd period, except 1, appears.

Keywords: Difference equations with periodic coefficients; elliptic curves; Lyness' type equations; QRT maps; rotation number; periodic orbits.

1. Introduction and Main Results

The study of periodic orbits in periodic nonautonomous discrete dynamical systems is a classical topic that, mainly driven by some conjectures in mathematical biology, has attracted again researchers interest in the last years. This is because they are good models for describing the dynamics of biological systems under periodic fluctuations whether due to external disturbances or effects of seasonality (see [Beyn *et al.*, 2008; Cushing & Henson, 2001, 2002; Elaydi & Sacker, 2005, 2006; Franke & Selgrade, 2003; Sacker & von Bremen, 2010], for instance, and references therein). On the other hand, the existence of discrete integrable systems with periodic coefficients is a topic of interest for the difference equations' community (see [Feuer *et al.*, 1996; Kulenović *et al.*, 2004] and also the references given below), and it is currently the focus of some mathematical physics research, specially on those systems belonging to the celebrated QRTfamily of maps introduced in [Quispel *et al.*, 1988, 1989] (see also [Duistermaat, 2010]), or on other

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