

On the Set of Periods of the 2-Periodic Lyness' Equation

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Abstract We study the periodic solutions of the *non-autonomous periodic Lyness' recurrence* $u_{n+2} = (a_n + u_{n+1})/u_n$, where $\{a_n\}_n$ is a cycle with positive values a, b and with positive initial conditions. Among other methodological issues we give an outline of the proof of the following results: (1) If $(a, b) \neq (1, 1)$, then there exists a value $p_0(a, b)$ such that for any $p > p_0(a, b)$ there exist continua of initial conditions giving rise to $2p$ -periodic sequences. (2) The set of minimal periods arising when $(a, b) \in (0, \infty)^2$ and positive initial conditions are considered, contains all the even numbers except 4, 6, 8, 12 and 20. If $a \neq b$, then it does not appear any odd period, except 1.

Keywords Difference equations with periodic coefficients · Elliptic curves · Lyness' type equations · QRT maps · Rotation number · Periodic orbits

1 Introduction

1.1 The Set of Periods

In this note we give a description of the *set of periods* of the 2-periodic Lyness' equations

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L. Alsedà i Soler et al. (eds.), *Difference Equations, Discrete Dynamical Systems and Applications*, Springer Proceedings in Mathematics & Statistics 180, DOI 10.1007/978-3-662-52927-0_22