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Limit cycles of polynomial differential equations with quintic homogeneous nonlinearities

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ABSTRACT

In this paper we mainly study the number of limit cycles which can bifurcate from the periodic orbits of the two centers

$$\dot{x} = -y, \qquad \dot{y} = x;$$
$$\dot{x} = -y(1-(x^2+y^2)^2), \qquad \dot{y} = x(1-(x^2+y^2)^2);$$

when they are perturbed inside the class of all polynomial differential systems with quintic homogeneous nonlinearities. We do this study using the averaging theory of first, second and third orders.

1. Introduction and statement of the results

In this paper we only consider differential equations in $\mathbb{R}^2$ of the form

$$\frac{dx}{dt} = P(x,y), \qquad \frac{dy}{dt} = Q(x,y), \tag{1}$$

where P and Q are polynomials of degree at most 5 with only homogeneous nonlinearities. We recall that a *limit cycle* of the differential equation (1) is a periodic orbit of this equation isolated in the set of all periodic orbits of Eq. (1).

The definition of limit cycles appeared in the years 1891 and 1897 in the works of Poincaré [15]. Almost immediately, in 1990, they become the main object to be studied in the statement of the second part of the 16-th Hilbert problem [9]. Later on van der Pol [16] in 1926, Liénard [11] in 1928 and Andronov [1] in 1929 shown that the periodic solution of a self-sustained oscillation of a circuit in a vacuum tube was a limit cycle in the sense defined by Poincaré. After this first observation of the existence of limit cycles in the nature, the existence, non-existence, uniqueness and other properties of the limit cycles have been intensively studied first by the mathematicians and the physicists, and more recently by the chemists, biologists, economists, etc. Nowadays the study of the limit cycles of the planar differential systems has been one of the main problems of the qualitative theory of the differential equations. See for instance the recent papers [18–20] and the references quoted there.

A *center* is a singular point of a differential system (1) for which there exists a neighborhood such that all the orbits in that neighborhood are periodic, with the exception of the singular point.

A good way of producing limit cycles is by perturbing the periodic orbits of a center. This technique has been studied intensively perturbing the periodic orbits of the centers, mainly of centers of the quadratic polynomial differential systems; see for instance the book of Christopher and Li [6], and the references therein.

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