Journal of Computational and Applied Mathematics 313 (2017) 273–283

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Centers and limit cycles of polynomial differential systems of degree 4 via averaging theory

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ARTICLE INFO

Article history:
Received 27 January 2016
Received in revised form 1 July 2016

MSC:
primary 34C15
34C25

Keywords:
Center
Limit cycle
Averaging method
Phase portrait
Generalized Kukles system

ABSTRACT

In this paper we classify the phase portraits in the Poincaré disc of the centers of the generalized class of Kukles systems

$$\dot{x} = -y, \quad \dot{y} = x + ax^3y + bxy^3,$$

symmetric with respect to the y-axis, and we study, using the averaging theory up to sixth order, the limit cycles which bifurcate from the periodic solutions of these centers when we perturb them inside the class of all polynomial differential systems of degree 4.

1. Introduction and statement of the main results

Two of the classical and difficult problems in the qualitative theory of polynomial differential systems in $\mathbb{R}^2$ is the characterization of their centers, and the study of the limit cycles which can bifurcate from their periodic orbits when we perturb them inside some class of polynomial differential equations.

Our work is related with the class of polynomial differential systems of the form

$$\dot{x} = -y, \qquad \dot{y} = x + Q_n(x, y), \tag{1}$$

having a center at the origin, where $Q_n(x, y)$ is a homogeneous polynomial of degree n, and in the study of the number of limit cycles which bifurcate from the periodic orbits of these centers when they are perturbed inside the class of all polynomial differential systems of degree n.

Differential polynomial systems (1) were called *Kukles homogeneous systems* in [1], see also [2,3]. The centers of systems (1) started to be studied by Volokitin and Ivanov in [4].

For $n = 1$ the differential systems (1) are linear, they can have centers, but the perturbation of these centers inside the class of linear differential systems cannot produce limit cycles, because it is well known that linear differential systems cannot have isolated periodic solutions in the set of all periodic solutions.

For $n = 2$ the phase portraits of system (1) symmetric with respect to the y-axis are a particular class of quadratic centers, and these are well studied, see [5].

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http://dx.doi.org/10.1016/j.cam.2016.08.047