Journal of Nonlinear Modeling and Analysis
Volume 1, Number 2, June 2019, 167–177
http://jnma.ca; http://jnma.ijournal.cn
DOI:10.12150/jnma.2019.167

Periodic Solutions of a Class of Duffing Differential Equations*

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Abstract In this work we study the existence of new periodic solutions for the well knwon class of Duffing differential equation of the form $x'' + cx' + a(t)x + b(t)x^3 = h(t)$, where c is a real parameter, $a(t)$, $b(t)$ and $h(t)$ are continuous T–periodic functions. Our results are proved using three different results on the averaging theory of first order.

Keywords Periodic solution, averaging method, Duffing differential equation, bifurcation, stability.

MSC(2010) 34C15, 34C25.

1. Introduction and statement of the main result

Several classes of Duffing differential equations have been investigated by many authors. They are mainly interested in the existence of periodic solutions, in their multiplicity, stability and bifurcation. See the survey of J. Mawhin [12] and for example the articles [2–4, 6, 9, 10, 13, 16, 18, 19].

In this work we shall study the class of Duffing differential equations of the form

$$x'' + cx' + a(t)x + b(t)x^3 = h(t), \tag{1.1}$$

where $c > 0$ is a constant, and $a(t)$, $b(t)$ and $h(t)$ are continuous T–periodic functions. These differential equations were studied by Chen and Li in the papers [2,3]. These authors studied the periodic solutions of equation (1.1) with the following additional conditions, either $b(t) > 0$, $h(t) > 0$ and $a(t)$ satisfies

$$a(t) \leq \frac{\pi^2}{T^2} + \frac{c^2}{4}, \quad \text{and} \quad a_0 = \frac{1}{T}\int_0^T a(t)dt > 0; \tag{1.2}$$

or $a(t) = a > 0$, $b(t) = 1$ and $c > 0$, a, c constants.

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*This work is supported by the Ministerio de Economía, Industria y Competitividad, Agencia Estatal de Investigación grants MTM2016-77278-P (FEDER) and MDM-2014-0445, the Agència de Gestió d'Ajuts Universitaris i de Recerca grant 2017SGR1617, and the H2020 European Research Council grant MSCA-RISE-2017-777911.