CROSSING LIMIT CYCLES OF PLANAR PIECEWISE LINEAR HAMILTONIAN SYSTEMS WITHOUT EQUILIBRIUM POINTS

REBIHA BENTERKI[1] AND JAUME LLIBRE[2]

ABSTRACT. In this paper we study the existence of limit cycles of planar piecewise linear Hamiltonian systems without equilibrium points. First we prove that if these systems are separated by a parabola they can have at most two crossing limit cyles, and if they are separated by a hyperbola or an ellipse they can have at most three crossing limit cycles. Additionally we prove that these upper bounds are reached. Second we show that there is an example of two crossing limit cycles when these systems have four zones separated by three straight lines.

1. Introduction

The problem of existence of limit cycles is one of the most and difficult problem in the qualitative theory of differential systems in the plane. Limit cycles appear in natural way in many appliations.

We recall that a *limit cycle* is a periodic orbit of a differential system which is isolated in the set of all periodic orbits of the system.

Recently the problem of existence and the number of limit cycles has also been studied for discontinuous piecewise linear differential systems, this study goes back to Andronov et all [1], and still have attention by researchers, mainly due to their simplicity and to their applications to a large number of phenomena, such as switches in electronic circuits, see for instance [2, 10, 11]. Lum and Chua [15] conjectured that a continuous planar piecewise linear system with two zones separated by a straight line can exhibit at most one limit cycle. Freire et al [5] proved this conjecture in 1998. For the planar discontinuous piecewise linear systems, Han and Zhang [7] conjectured that these systems can have at most two crossing limit cycles when we separate them by a straight line, but Huan and Yang [8] gave a numerical example with three limit cycles, this result was proved analytically by Llibre and Ponce [13]. In 2015 Llibre et all [12] proved that if we separate the planar discontinuous piecewise linear differential centers by a straight line we can not have any limit cycle. Recently, in [3, 9, 14] were studied planar discontinous linear differential centers separated by an algebraic curve, such that a conic, or a reducible and irreducible cubic, and it was proved that these differential systems can exhibit at most three crossing limit cycles having two intersection points with

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