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Dynamics of the polynomial differential systems with homogeneous nonlinearities and a star node

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ABSTRACT

We consider the class of polynomial differential equations $\dot{x} = \lambda x + P^n(x, y)$, $\dot{y} = \lambda y + Q^n(x, y)$, in $\mathbb{R}^2$ where $P^n(x, y)$ and $Q^n(x, y)$ are homogeneous polynomials of degree $n > 1$ and $\lambda \neq 0$, i.e. the class of polynomial differential systems with homogeneous nonlinearities with a star node at the origin.

We prove that these systems are Darboux integrable. Moreover, for these systems we study the existence and non-existence of limit cycles surrounding the equilibrium point located at the origin.

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1. Introduction and statement of the main results

By definition a two dimensional *polynomial differential system* in $\mathbb{R}^2$ is a differential system of the form

$$\frac{dx}{dt} = \dot{x} = P(x, y), \qquad \frac{dy}{dt} = \dot{y} = Q(x, y), \tag{1}$$

where the dependent variables x and y, and the independent one (the time) t are real, and $P(x, y)$ and $Q(x, y)$ are polynomials in the variables x and y with real coefficients. We denote by $m = \max\{\deg P, \deg Q\}$ the *degree* of the polynomial system.

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