COUNTEREXAMPLE TO KALMAN AND MARKUS-YAMABE CONJECTURES IN LARGER THAN 3

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Abstract. We construct a system that is globally Hurwitz with a periodic orbit. With this system we build a counterexemple to Kalman conjecture in dimension 4, and consequently to Markus-Yamabe conjecture. Then we extend these counterexamples to dimension larger than 4.

AMS (MOS) Subject classification: 34C35, 34H05

1 Introduction

A matrix is Hurwitz if all its eigenvalues have negative real part. A system is Hurwitz if its Jacobian is Hurwitz in every point of \mathbb{R}^n .

Consider the system of ordinary differential equations x'(t) = F(x(t)), where F(x) is C^1 and x = 0 is an equilibrium point. If DF(0) is Hurwitz then by Hartman-Grobman Theorem [8] the origin is a local asymptotically stable solution. The question is what hypothesis we have to add to F(x) to assure that the origin is a global attractor. In 1960 Markus and Yamabe [16]