Rabbits, Basilicas, and Other Julia Sets Wrapped in Sierpinski Carpets

Paul Blanchard, Robert L. Devaney, Antonio Garijo, Sebastian M. Marotta, Elizabeth D. Russell

1 Introduction

In this paper we consider complex analytic rational maps of the form

$$F_\lambda(z) = z^2 + c + \frac{\lambda}{z^2}$$

where $\lambda, c \in \mathbb{C}$ are parameters. For this family of maps, we fix c to be a parameter that lies at the center of a hyperbolic component of the Mandelbrot set, i.e., a parameter such that, for the map

$$F_0(z) = z^2 + c,$$

0 lies on a periodic orbit. We then perturb F_0 by adding a pole at the origin. Our goal is to investigate the structure of the Julia set of F_λ, which we denote by $J(F_\lambda)$, when λ is nonzero.

For these maps, the point at ∞ is always a superattracting fixed point, so we have an immediate basin of attraction of ∞ that we denote by B_λ. As a consequence, we may also define the filled Julia set for these maps to be the set of points whose orbits remain bounded. We denote this set by $K(F_\lambda)$.

In the case where c is chosen so that the map has a superattracting cycle of period 1, the structure of $J(F_\lambda)$ has been well-studied [1], [3], [4], [5]. In this case, $c = 0$ and the map is $z^2 + \lambda/z^2$. This map has four "free" critical points at the points $c_\lambda = \lambda^{1/4}$, but there is essentially only one critical orbit, since one checks easily that $F_\lambda^2(c_\lambda) = 4\lambda + 1/4$. Hence all four of the critical orbits land on the same point after two iterations. Then the following result is proved in [1].

Theorem. *Suppose the free critical orbit of $z^2 + \lambda/z^2$ tends to ∞ but the critical points themselves do not lie in B_λ. Then the Julia set of this map is a Sierpinski curve. In particular, there are infinitely many disjoint*