IOP PUBLISHING NONLINEARITY

Nonlinearity **25** (2012) 2615–2624 doi:10.1088/0951-7715/25/9/2615

A note on the Darboux theory of integrability of non-autonomous polynomial differential systems

David Blázquez-Sanz[1] **and Chara Pantazi**[2]

[1] Instituto de Matemáticas y sus Aplicaciones - Universidad Sergio Arboleda, Calle 74 No. 14-14, Bogotá, Colombia
[2] Departament de Matemàtica Aplicada I, Universitat Politècnica de Catalunya, (EPSEB), Av. Doctor Marañón, 44–50, 08028 Barcelona, Spain

E-mail: david@ima.usergioarboleda.edu.co and chara.pantazi@upc.edu

Received 26 December 2011, in final form 26 June 2012
Published 10 August 2012
Online at stacks.iop.org/Non/25/2615

Recommended by D V Treschev

Abstract
In this work, we unfold some differential algebraic aspects of Darboux first integrals of polynomial vector fields. An interesting improvement is that our approach can be applied both to autonomous and non-autonomous vector fields. We give a sufficient and necessary condition for the existence of a Darboux first integral of a specific form for a polynomial vector field with some known algebraic invariant hypersurfaces. For the autonomous case, the classical result of Darboux is obtained as a corollary. For the non-autonomous case our characterization improves a known criterium.

Mathematics Subject Classification: 34C14, 12H05

1. Introduction

One of the most important subjects in mathematical physics is the discussion of integrability of differential equations started by Newton in the 17th century. There is not an unified approach to integrability. There are many different techniques for integrating differential equations or proving their non–integrability within some specific framework: Lie symmetries [29], Noether symmetries [7], Lax pairs [17], Painlevé analysis [5], differential Galois theory [2, 27, 31] and Darboux's method [13], among others.

A useful approach is the search of invariants and their classification. The first class of invariants are the conserved quantities or first integrals. There are also some other invariants like Jacobi multipliers, tensor invariants or symmetries. The search and classification of those invariant objects of differential equations is still a very active research topic.

In this paper, we give some generalization of a classical method started by Darboux in 1878 for the computation of first integrals of polynomial differential systems [13, 14]. More

