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Existence of limit cycles in a tritrophic food chain model with Holling functional responses of type II and III

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We are interested in the coexistence of three species forming a tritrophic food chain model. Considering a linear grow for the lowest trophic species, Holling III and Holling II functional response for the predator and the top-predator, respectively. We prove that this model has stable periodic orbits for adequate values of its parameters.

Keywords: periodic orbit; Hopf bifurcation; population dynamics; Lyapunov coefficient

1. Introduction

The dynamic relationship between predators and their prey has long been studied and will continue being one of the dominant topics in both ecology and mathematical ecology because of its universality and importance, see for instance [1–4]. These problems may appear to be simple mathematically at first sight. In fact, they are often very challenging and complicated.

Although the predator–prey theory has seen much progress in the last 40 years, there are still mathematical and ecological problems unsolved. In [5], the authors considered a model for three species that compete for three resources, and they proved that the existence of two limit cycles evolves the coexistence equilibrium points. In [1], it is studied a model representing a tritrophic food chain composed of a logistic prey, a Holling type II predator, and a Holling type II top-predator. They proved using the averaging theory the existence of a stable periodic orbit contained in the domain of interest. In [6], the authors analyzed a model representing a tritrophic food chain composed by a prey with linear grow and a functional response Holling type III for the predator and the top-predator. They proved using the averaging theory the existence of three-dimensional parameter families for which the model has two equilibrium points of zero-Hopf type contained in the domain of interest and also prove the existence of a simultaneously double zero–Hopf bifurcation. In [7] the author's show the existence of a Boyacov bifurcation in a tritrophic food chain model.

In this paper, we analyze a tritrophic food chain model considering Holling functional response of type III for the predator, Holling type II for the top-predator, and linear grow for the prey.

A tritrophic food chain model with functional response $f(x)$ and $g(y)$ for the middle and the highest trophic species, respectively, has the form

$$\begin{aligned} \dot{x} &= \rho x - f(x)y, \\ \dot{y} &= cyf(x) - g(y)z - dy, \\ \dot{z} &= z\,(eg(y) - d_2)\,, \end{aligned} \tag{1}$$

where x represents the prey that gets eaten by a species of density y (mesopredator) that feeds the species with density z (super-predator). The parameters ρ, c, d, d_2, and e are positive constants.

The functional responses of the mesopredator $f(x)$ and the super-predator $g(x)$ satisfy

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