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Periodic solutions of linear, Riccati, and Abel dynamic equations

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ABSTRACT

We study the number of periodic solutions of linear, Riccati and Abel dynamic equations in the time scales setting. In this way, we recover known results for corresponding differential equations and obtain new results for associated difference equations. In particular, we prove that there is no upper bound for the number of isolated periodic solutions of Abel difference equations. One of the main tools introduced to get our results is a suitable Melnikov function. This is the first time that Melnikov functions are used for dynamic equations on time scales.

1. Introduction and main results

Consider the polynomial periodic differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = x' = a_0(t) + a_1(t)x + \cdots + a_{n-1}(t)x^{n-1} + a_n(t)x^n, \tag{1.1}$$

where $x, t \in \mathbb{R}$ and $a_0, a_1, \ldots, a_n : \mathbb{R} \to \mathbb{R}$ are smooth ω-periodic functions. The question of studying its number of ω-periodic solutions in terms of n was proposed by N. G. Lloyd (see [20]) and C. Pugh (see [19]). Notice that (1.1) with $n = 1$ (resp. $n = 2$) is a *linear equation* (resp. a *Riccati equation*), and it is well known that linear (resp. Riccati) equations have either a continuum of periodic solutions or at most 1 (resp. 2) periodic solutions, see for instance [19,21]. When $n = 3$, (1.1) is called *Abel equation*, and the more relevant result on it was proved in [19]: For any k, there exist equations of the form (1.1), with a_i trigonometric ω-periodic polynomials, having at least k isolated ω-periodic solutions. A similar result holds for $n > 3$. The results for $n = 3$ have interest by themselves, but also because Abel equations appear when

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